SHORE A DUROMETER AND ENGINEERING PROPERTIES

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Presented at the Fall Technical Meeting of
The New York Rubber Group
Thursday, September 24, 1998

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What Is Durometer?

**Schematic:**

![Durometer Schematic](image)

**Surface Hardness Indicator**

A durometer gage measures the depth of surface penetration for a pin of a predetermined geometry. The device is essentially a machine shop dial indicator measuring deflections between 0.000 and 0.100 inches; the gage reads “0” at 0.100 inches penetration and “100” at 0.000 inches penetration. A spring with a known (also predetermined) stiffness is used. In this presentation, International Rubber Hardness Degrees (IRHD) and durometer are used interchangeably, since the test methods and theory are basically the same. Note, however, that durometer and IRHD will not necessarily be numerically equal.

What Is It Meant To Measure?

**Material Stiffness/Young’s Modulus, Shear Modulus**

The depth of penetration is directly related to the force required, thus durometer is typically used as a stiffness indicator (since both force and deflection are measured). This would seem to be a logical definition, since the relationship between force and deflection usually derives stiffness.

Remember, however, that rubber is a non-linear material (i.e. the uniaxial stress vs. strain curve does not form a straight line).

**Shear Modulus, G**

Slope of shear stress strain curve is known as the shear modulus, G. Figure 2 shows a schematic of shear loading. Based on this geometry, the stress and strain are calculated as follows:
Figure 2: Shear Loading and Deflection

Shear Stress = \( \frac{F}{A} \)

Shear Strain = \( \frac{\delta}{l} \)

The resulting stress vs. strain curve for a material with \( G = 1,500 \) psi looks like that of Figure 3.

Figure 3: Stress Strain Curve for Shear
Define Young’s Modulus, $E$:
In terms of linear materials, “$E$” is the slope of the uniaxial stress-strain curve. For rubber, it is derived in terms of the shear modulus. Due to the fact that rubber is incompressible, $E=3G$.

Why is it Used?

Design and Analysis
Aside from permeability issues, contact pressure (or stress) due to deformation is what causes a rubber element to seal. The seal is maintained only if the rubber material remains intact both mechanically and chemically (realize that some mechanical changes are the results of chemical changes). This is why stress relaxation and compression set are key properties when considering the longevity of a seal. These values are functions of the stiffness of the material (and are time dependent). When considering a seal design, material stiffness is extremely important. The stiffness, or Young’s modulus (defined loosely), is a measure of how much spring force a rubber component will exert when subjected to a deformation. Engineers need a “quick and dirty” way to estimate material stiffness when designing rubber components.

Quality Control
Stiffness must be controlled, since variations in stiffness mean variations in contact and peak stresses. As some of you may know, over-compressing a seal can be as bad as under compressing. Thus, stiffness is crucial to seal design.

Ease of Measurement
Durometer gages are relatively inexpensive, and do not require an intensive amount of training to operate. Basically, it’s cheap and easy.

What’s the Problem?

Operator Dependent
Not many lab technicians use a stopwatch to count out seconds before measuring. If you have ever performed a durometer measurement, you have noticed that the gage drops with time.

Linear Instrument Measures Using a Non-Linear Method
We previously mentioned that rubber is a non-linear material. For example, let’s say material “A” has Modulus $E$, while material “B” has Modulus $E/2$; the measured durometer for material “A” is not twice that of material “B.” That is, twice the durometer is not twice the stiffness (and vice versa). Depth of penetration is not a linear function of
the stiffness of the material. Figure 4 shows a chart of measured IRHD versus Young’s Modulus. Note that the chart uses a semi-log scale (IRHD is on a linear scale plotted against the logarithm of Young’s Modulus).

[Image of Figure 4: IRHD vs. Log "E"]

**False Sense of Security**

Stiffness is typically controlled using durometer measurements. Quality Control is not necessarily achieved using durometer measurements. These measurements allow for large variations in modulus, although appearing tightly controlled.

Look at Figure 5 below, for example. The graph shows the same information as in Figure 4, but the axes have been switched and the data are shown on linear scales. Let’s suppose that a customer specifies that a compound must have an IRHD of 70. The dotted line at 70 shows the nominal specification, and the two solid lines show the allowances. The nominal stiffness is about 750 psi. The material stiffness at the low and high ends of the tolerance is 650 psi and 1,000 psi respectively. Translated into psi, the material specification allows a modulus of 750 psi+250psi/-100 psi.

As a second example, let’s suppose a customer specifies a durometer (or IRHD) of 90. According to Figure 6, the nominal stiffness is 2,600 psi. The tolerances given allow for stiffness between 1,800 psi and 4,700 psi. Thus, although the durometer measurements may appear to be consistent and tightly controlled, the stiffness is allowed to have quite large variation, while still “meeting” the customer’s specifications.
Figure 5: Young's Modulus vs. IRHD, 70 Durometer Nominal

Figure 6: Young's Modulus vs. IRHD, 70 and 90 Nominal Durometer
What Do We Do About This?

**Focus the Scope**

Think about the measurement system in terms of sensitivity. Sensitivity of a measurement device/method is the change in signal for a given change in the property to be measured. For example, a manufacturer of tensile test machines may report the sensitivity of a load cell as 0.5 Volts per 1 pound force over the entire range of the load cell. A load cell should double its output voltage if the applied force doubles.

We want to find an easy way to approximate Young’s Modulus. The measurement method should have a constant “sensitivity” over the entire range of interest, and it should be easy to calculate.

**Define Strain Energy Density**

Strain Energy Density is defined as the energy stored (per unit volume) as a result of applied deformations (strains). Think about a rubber band: after you have stretched it, it has stored energy that is recoverable. Consider two rubber bands of different thickness, both of the same length. Suppose we stretch both of them to twice their original length, or 100% elongation. The thicker band will require more energy. This is why we use strain energy density: dividing by the volume allows us to compare specimens of different thickness/geometry. Strain energy density is calculated as the area under the stress vs. strain curve. To calculate the strain energy density at 20% elongation, we would find the area under the curve only up to 20% stretch as shown in Figure 7.

![Figure 7: Calculation of Strain Energy Density at 20% Elongation](image-url)
Material Model: Neo-Hookean Materials

S.E.D. for Neo-Hookean Material:

By a derivation we will not get into here, if we define the strain energy density in terms of one material constant, G, we have:

\[ W = \frac{G}{2} \left( \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \right) \]

W=Strain Energy Density
G=Shear Modulus
\( \lambda_i \)=Principle Stretches (Current Length over Initial Length)

We can also say that since rubber is incompressible, the following relationship will be true for uniaxial tension or compression:

\[ \lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda_1}} \]

Substituting back into the equation for strain energy density, we have:

\[ W = \frac{G}{2} \left( \lambda_1^2 + \frac{2}{\lambda_1} - 3 \right) \]

Thus, to calculate shear modulus (or Young’s Modulus using E=3G) we can use the measured strain energy density at 20% elongation. Once we have tested a simple dog-bone type specimen (ASTM D 412), the only unknown in the above equation is the shear modulus, G. We can integrate the stress vs. strain curve up to 20% to get “W”, and since we integrated up to 20% elongation, lambda is equal to (1.2). Solving for “G” gives:

\[ G \cong (18.75) \cdot W_{20\%} \]

* For a range of strains up to about 60% (or equivalently, \( \lambda=1.6 \)).
The Pay-Off (At Last!)

Linear Sensitivity

The strain energy density at 20% elongation is linearly sensitive to changes in Modulus where the durometer measurement is very sensitive at low hardness and is almost independent of Modulus at higher hardness. This can be shown by graphing Durometer vs. Modulus and $W_{20\%}$ vs. modulus curves, as in Figure 8 below.

![Comparison of IRHD and W_{20\%}](image)

**Figure 8: Comparison of IRHD and W_{20\%}**

Another interesting way to compare sensitivity of durometer and strain energy density measurements is to graph the derivative (or slope) of the curves shown in Figure 8. Figure 9 shows the sensitivity of each measurement type. Note that as durometer increases, the sensitivity of durometer measurements goes to zero (while the response of strain energy density is flat).
Automation is Easy

Most tensile testing machines can automatically calculate strain energy density after each test is completed. Some newer software will even enter the data directly into statistical process control programs—without user intervention. Since the speed of a tensile test is controlled by computer, variation due to the operator is minimized.

Conclusion

**Durometer**

Although durometer measurements may be cheap and easy, control of durometer does not assure control of material stiffness. The use of such instruments will continue based on tradition (we’ve always done it this way) and because the measurements are easy to take.

**Strain Energy Density**

Strain energy density can be a useful design tool, and can also help assure tight quality control. This type of calculation should be incorporated into rubber laboratories as a standard report item.
Additional Notes:

According to Neo-Hookean Hyperelasticity:

\[ W = \frac{G}{2} \left( \lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \right) \]

Where \( G = \) Shear Modulus and 
\( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are Principle Stretch Ratios

The applied stretch is:
\( \lambda_1 = 1 + \varepsilon_{11} \)

For simple extension (due to incompressibility), we have the relation:

\[ \lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda_1}} \]

\[ \sigma_{Tension} = \frac{\partial U}{\partial \lambda} = G \left( \lambda - \frac{1}{\lambda^2} \right) \]
References

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Peacock, Christopher. “Quality Control Testing of Rubber Shear Modulus”,